For every first-order ODE, try one of the following ways of solving: separation of variables, exact integration, substituting $y = vx$ in homogeneous equations, using integrating factors to make linear equations exact.

1. Solve the differential equation: $x\ dy/dx = 2(y-4)$. What does the family of solution curves look like?

2. Is the following ODE exact? Why/why not? Solve this ODE.

$$(\ln x)dy + \frac{y}{x}dx = 0. \tag{1}$$

3. Solve the damped harmonic oscillator equation

$$m\ddot{x} = -kx - \gamma\dot{x} \tag{2}$$

using the techniques for solving a second order ODE. What is the nature of the solution at high and low values of the damping coefficients γ?

4. (a) Convert the same second order damped harmonic oscillator equation to a first order ODE in the phase space $(x, \dot{x})$. What is the general form of the solution? Your answer should involve a matrix exponential.

   (b) You will learn how to efficiently exponentiate such matrices in your quantum mechanics course. For now, investigate the behavior of the oscillator in the phase space in the no-damping limit $\gamma = 0$ and in the weak spring limit, $k = 0$. For solving this examine the first five terms in the Taylor expansion of the exponential after taking each limit. You will also need the Taylor expansions

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... \tag{3}$$

and

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - ... \tag{4}$$

From your solved expressions for $x(t)$ and $v(t)$, visualize the motion in phase space (x, v) by evaluating and plotting it at several different values of t, in the Desmos calculator.

5. Solve the differential equations

   (a) $x^2dy + y(y-x)dx = 0$, given that $y(x=1) = 1$.

   (b) $dy/dx = y - x$, the ODE which you numerically approximated using Euler method.

   (c) $x\ dy/dx - 2y = x^4 \sin x$.