1. Write $g(x) = x/\pi$, defined as a periodic function within $x \in [-\pi, \pi)$, as a real Fourier series with sines and cosines. Make use of the orthogonality of the sines and cosines basis, and the fact that $g(x)$ is an odd function. A plot of $g(x)$, the 'sawtooth wave', is here.

   Use Desmos, an online graphing calculator, to plot successive Fourier approximations to $g(x)$.

   The orthogonality relations for the Fourier basis are as follows for $m, n \neq 0$,

$$\int_{-\pi}^{\pi} dx \sin(mx)\sin(nx) = \begin{cases} \pi & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases} \tag{1}$$

$$\int_{-\pi}^{\pi} dx \cos(mx)\cos(nx) = \begin{cases} \pi & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases} \tag{2}$$

$$\int_{-\pi}^{\pi} dx \sin(mx)\cos(nx) = 0 \tag{3}$$

$$\int_{-\pi}^{\pi} dx \sin(mx) = \int_{-\pi}^{\pi} dx \cos(mx) = 0. \tag{4}$$

2. Let $f(t)$ be a periodic function with the properties $f(-t) = -f(t)$ and $f(t+\pi) = -f(t)$ for all t. Which Fourier coefficients are nonzero?

3. Obtain the Fourier transform of the Gaussian function

$$f(t) = e^{-at^2}. \tag{5}$$

   How does the Fourier transform change graphically when a is increased? Use Desmos to plot.

4. Define the *convolution* of two functions f and g as

$$h(t) \equiv (f * g)(t) \equiv \int_{-\infty}^{\infty} f(t-t')g(t')dt'. \tag{6}$$

   Express the Fourier transform of h in terms of the Fourier transforms of f and g.

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