Eigensystems

A spin-1 nucleus is placed in the presence of a strong, external magnetic field. When the field is present, there are three possible orientations of the spin on the nucleus that are eigenstates of the external field: parallel to the field, orthogonal to the field, or antiparallel to the field. We will represent these three states with the vectors $|1\rangle$, $|0\rangle$, and $|-1\rangle$, respectively. We now apply an additional field, represented by the operator $\hat{A}$, that has the following effect on each of these initial eigenstates:

$$\begin{aligned} \hat{A}\,|-1\rangle &= \frac{1}{\sqrt{2}}\big(\,|-1\rangle + |1\rangle\,\big) \\ \hat{A}\,|0\rangle &= |0\rangle \\ \hat{A}\,|1\rangle &= \frac{1}{\sqrt{2}}\big(\,|-1\rangle - |1\rangle\,\big) \end{aligned}$$

For each of the problems below, assume that the states $|1\rangle$, $|0\rangle$, and $|-1\rangle$ are orthonormal.

1. Construct the matrix representation of the operator $|A\rangle$ in the basis indicated.
2. If this matrix **A** has any eigenvalues/eigenvectors that you can determine by inspection, *i.e.* without diagonalizing a matrix, do so to simplify the remainder of the problem.
3. Determine the remaining eigenvalues/eigenvectors for this matrix.

Summation Notation

4. Show that the trace of the matrix product **AB** is equal to the trace of the matrix product **BA** for any two square matrices **A** and **B** of (the same) arbitrary size.