Eigensystems

A spin-1 nucleus is placed in the presence of a strong, external magnetic field. When the field is present, there are three possible orientations of the spin on the nucleus that are eigenstates of the external field: parallel to the field, orthogonal to the field, or antiparallel to the field. We will represent these three states with the vectors $|1\rangle$, $|0\rangle$, and $|-1\rangle$, respectively. We now apply an additional field, represented by the operator \hat{A} , that has the following effect on each of these initial eigenstates:

$$\hat{A} |-1\rangle = \frac{1}{\sqrt{2}} (|-1\rangle + |1\rangle)$$

$$\hat{A} |0\rangle = |0\rangle$$

$$\hat{A} |1\rangle = \frac{1}{\sqrt{2}} (|-1\rangle - |1\rangle)$$

For each of the problems below, assume that the states $|1\rangle$, $|0\rangle$, and $|-1\rangle$ are orthonormal.

- 1. Construct the matrix representation of the operator $|A\rangle$ in the basis indicated.
- 2. If this matrix \mathbf{A} has any eigenvalues/eigenvectors that you can determine by inspection, *i.e.* without diagonalizing a matrix, do so to simplify the remainder of the problem.
- 3. Determine the remaining eigenvalues/eigenvectors for this matrix.

Summation Notation

4. Show that the trace of the matrix product **AB** is equal to the trace of the matrix product **BA** for any two square matrices **A** and **B** of (the same) arbitrary size.

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