Orthonormal Basis Vectors

1. What pairs of orthogonal vectors can be formed from the set of vectors below?

$$\begin{bmatrix}2\\3\end{bmatrix} \qquad \begin{bmatrix}i\\2-i\end{bmatrix} \qquad \begin{bmatrix}-3i\\4\end{bmatrix} \qquad \begin{bmatrix}0\\i\end{bmatrix} \qquad \begin{bmatrix}2i-1\\-1\end{bmatrix}$$

2. What is the span of the each of these pair(s) of vectors?

3. Normalize the vectors in all pair(s) of vectors you found in (1) above.

4. Write each vector in (1) that was *not* orthogonal to any other vectors as a linear combination of one of these normalized pair(s) of vectors.

Function Spaces

5. For this problem, we will be considering functions defined over the region $x \in [0, 1]$. Mathematically, we will have

$$f(x) = \begin{cases} g(x) & x \in [0,1] \\ 0 & \text{otherwise} \end{cases},$$

where $g(x)$ is some other function. Within this function space, we'll work with basis functions defined according to

$$g_k(x) = \sin(k\pi x) \qquad k = 1, 2, 3, \ldots.$$

(a) What is the definition of the inner product for this function space?

(b) Choose any two functions from this set and show that they are orthogonal.

(c) Normalize the function corresponding to $k = 2$.