1. Evaluate μ and σ^2 for the following distributions:

   (a) You have four cards numbered 1 through 4. You draw two cards at random and find the sum of numbers on the two cards, which is a random variable X.

   (b) A Poisson distribution, which is a discrete probability distribution given by $P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$, where λ is a parameter of the distribution, and $x = 0, 1, 2, \ldots, \infty$.

2. You have an unbiased 4-sided die of colors: red, yellow, blue, green. You roll the die N times. Let N_B be the number of times you roll blue.

   (a) What is the probability of observing a specific sequence of colors?

   (b) How many ways are there to roll a sequence that has exactly N_B blue rolls?

   (c) What is the probability of observing N_B blue rolls when rolling the die N times?

3. You perform an experiment four times, obtaining four data points: $\{5, 7, 1, 3\}$. Over the next two days, you repeat the experiment again, obtaining two more sets of data: $\{1, 1, 2, 3\}$ and $\{8, 2, 4, 3\}$. You suspect the underlying random variable your experiment is sampling data has mean $\mu = 3$ and variance $\sigma^2 = 3$, but you want to confirm if this is true.

   (a) What is the mean of the sample data ($\mu_{\bar{X}_n}$), according to your current data? Hint: what's your sample size (n) here?

   (b) Over the course of your PhD, you proceed to repeat this same experiment an infinite number of times, always being careful to obtain data sets containing four points. How do you expect the sample mean ($\mu_{\bar{X}_4}$) and variance ($\sigma^2_{\bar{X}_4}$) to change?

   (c) You're excited to present these results to your colleagues and are making a plot of your data. Describe in words how the data you collected corresponds to a plot.

   (d) Challenge: At some point in your third year of grad school, you continue collecting data on this same experiment, but sometimes your dataset has $n = 4$ points, and sometimes it has more or less, depending on how productive you're feeling that day. Can you incorporate the datasets for which $n \neq 4$ into your original analysis?

4. The uniform distribution is one of the simplest probability distributions, where the probability of any value x between a and b is equal. Find $p(x)$, μ, and σ^2 for this distribution.

5. The following function is a probability distribution, where N is the normalization constant:

$$p(x) = N \exp\left[-\frac{x^2 + ax}{2\sigma^2}\right]. \tag{1}$$

Find the normalization constant and then find the mean and variance of the distribution.

6. Evaluate the following integrals:

   (a) $I = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx dy e^{-\left(x^2+y^2\right)}$

   (b) $I = \int_{3}^{\infty} dx \exp\left(-\frac{(x-3)^2}{2\sigma^2}\right)$